Alexander V. Kislov[1]*, Polina A. Morozova[2]
[1] Lomonosov Moscow State University, Faculty of Geography, Department of Meteorology and Climatology; Leninskie Gory, 1, 119991 Moscow, Russia.
*** Corresponding Author;** e-mail: avkislov@mail.ru
[2] Russian Academy of Science, Institute of Geography; Staromonetny, 29, Moscow, 119017, Russia. E-mail: morozova_polina@mail.ru

THE GROSSER ALETSCHGLETSCHER DYNAMICS: FROM A "MINIMAL MODEL" TO A STOCHASTIC EQUATION

ABSTRACT. Mountain glaciers manifest oscillations at different time-scales. Apart from synchronous reaction to lasting changes, there is asynchronism between climatic forcing and observed anomalies of the glaciers. Based on general theories on the laws of temporal dynamics relating to massive inertial objects, the observed interannual changes of glacier length could result from the accumulation of small anomalies in the heat/water fluxes. Despite the fact that the original model of the dynamics of mountain glaciers is deterministically based on the physical law of conservation of water mass, the model of length change is interpreted as stochastic; from this perspective, it is the Langevin equation that incorporates the action of temperature anomalies and precipitation like random white noise. The process is analogous to Brownian motion. Under these conditions, the Grosser Aletschgletscher (selected as an example) is represented by a system undergoing a random walk. It was shown that the possible range of variability covers the observed interval of length fluctuations.

KEY WORDS: glacier dynamics, climate change, the Grosser Aletschgletscher, the Langevin equation.

INTRODUCTION

Mountain glaciers demonstrate oscillations at different time scales. Apart from synchronous reaction to lasting changes, observed anomalies of glacier length and other parameters could be derived from accumulation of small heat/water flux anomalies of opposite signs. Their residual effect forms a response much like a random walk. This effect reflects the general mechanism of functioning of "massive" inertial objects under the influence of noise, which follows from the concept of Brownian motion [Gardiner, 1996]. Note that this assumption cannot be precisely proven. However, it is possible to determine whether the results of observations contradict or agree with this hypothesis.

A glacier model should be based on three-dimensional dynamics of a viscous-plastic body (see e.g. [Cuffey & Paterson, 2010]). However, this approach is not yet effective. Specific information about the parameters of a glacier (which vary for various glaciers) and features of underlying rocks and relief are not often available. This led to the development of simple, integrated models of glaciers [Oerlemans, 2008; Harrison, 2013].

The study of this problem is within the framework of the mathematical "minimal model" of the dynamics of mountain glaciers [Oerlemans, 2008]. However, the absence of needed parametrizations restricts across-the-broad application and, thus, only one glacier was studied. It is the Grosser Aletschgletscher, located in the Bernese Oberland region, Switzerland. The principle position of the theory [Oerlemans, 2008] is that a glacier has always an equilibrium profile due to the balance of mass budget variations and movement of ice. It means, in particular, that the length (L) and mean ice thickness (H_m) are linked as follows: $H_m = \alpha\sqrt{L}/(1+\mu\nu)$ (see [Cuffey & Paterson, 2010] for the explanation of the parabolic form of this function), where α, μ, and ν are parameters of the glacier. This simple function between glacier length and ice thickness is derived by assuming perfect plasticity, which is not fully appropriate when dealing with short-term variations in the glacier front position. Under such conditions, the mass conservation equation is transformed into an equation describing the change of length of the glacier [Oerlemans, 2008]:

$$\frac{dL}{dt} = \frac{2(1+\mu\nu)}{3\alpha}\left(-0.5\beta\nu L^{3/2} + \frac{\alpha\beta}{1+\mu\nu}L + \beta(b_0 - E)L^{1/2}\right). \quad (1)$$

It was additionally assumed that the slope of the bed is constant (ν) (β_0 is the altitude of the upper point of the glacier) and mass balance is described by a linear function of the height (h): $B = \beta(h - E)$, where E is the equilibrium-line altitude and β is the parameter of the glacier. Mass balance is approximated by a linear function which is far simpler than it is actually observed on glaciers, but several glaciers demonstrate this type of linear dependence [Kunachovich et al, 1996; Oerlemans, 2008; Fluctuations...].

MATERIALS AND METHODS

As previously proposed, the origin of long-lasting glacier anomalies could be interpreted based on the theory of Brownian motion. It argues that multi-scale stochastic dynamics of a system is formed through interaction of its fast and slow components. Positive and negative fast anomalies do not cancel each other out and their residual effects accumulate slowly to form a large deviation from the initial state. However, negative feedbacks, which usually exist in the system, prohibit large deviations; therefore, the steady state regime of slow chaotic oscillations is being realized. K. Hasselmann [1976] used this mechanism to describe and reproduce the stochastic behavior of several geophysical processes.

The random time evolution of a Brownian particle position (one-dimensional) $x = x(t)$ approximately satisfies the Langevin equation [Gardiner, 1996]:

$$\frac{dx}{dt} = -\omega x(t) + \vartheta(t). \quad (2)$$

Here, ω^{-1} is characteristic time of the system. Autocorrelation function of the force $\vartheta(t)$ is approximated by:

$$\langle \vartheta(\xi)\vartheta(\zeta)\rangle = \sigma^2 \exp\left(-\frac{|\xi - \zeta|}{\tau_\vartheta}\right), \quad (3)$$

where ξ and ζ are different moments and <...> denotes the ensemble average value. The acute form of the autocorrelation curve is assumed to be approximated by the δ-function curve. It is consistent with the property of the discussed physical problem, $\tau_\theta << \omega^{-1}$. As a result, a time series of $x = x(t)$ is expressed by the Ornstein-Uhlenbeck process [Gardiner, 1996].

The solution of the task depicted by equations (2) and (3) shall be discussed later. Now let us emphasize that in order to simulate a stochastic process $x = x(t)$, parameters ω and τ_θ should be chosen. This can be done using two methods. The first one is a standard statistical method and allows us to evaluate the parameters based on observations. Examples of its successful utilization are presented in [Oerlemans, 2008].

In the second method, deterministic equations reflecting conservation laws ("first principia") could be used for estimation of parameters of the equivalent stochastic model. This approach is much more reliable because it allows us to estimate parameters against a background change (e.g. climate change). This approach will be described below for investigation of the glacier length variations.

Let us develop the stochastic model based on the deterministic equation (1) which will be reduced to (2). Let us emphasize that the "forces" disturbing the dynamics of the glacier, fluctuate very rapidly compared to its slow response.

Dividing both sides of equation (1) by $\sqrt{L}$ and using $y=\sqrt{L}$, we have:

$$\frac{dy}{dt}=-\frac{(1+\mu\nu)\beta\nu}{6\alpha}y^2+\frac{\beta}{3}y+ \\ +\frac{(1+\mu\nu)\beta(b_0-E)}{3\alpha}. \quad (4)$$

It is a Riccati equation, which can be simplified using the linearization procedure. It can be done with confidence because change of L is typically small compared to its averaged value L_0. Indeed, it equals to only 6 % for the Grosser Aletschgletscher, 8 % for the Nigardsbreen Glacier, 4 % for the Glacier de Bosson, 5 % for the Brikdalsbreen Glacier, 19 % for the South Cascade Glacier, etc. It should be emphasized that these data include not only fluctuations but a detected strong negative trend associated with global warming [Fluctuations...]. Thus, the simplified form of the equation is as follows:

$$\frac{d\Delta y}{dt}=-\lambda\Delta y+\eta, \quad (5)$$

here, η is defined as $\eta = -cE_0\Delta E$, where E_0 is scale of variation of the equilibrium-line altitude, and ΔE is a dimensionless value; therefore, $\Delta E \approx 1$. Other coefficients are denoted using the parameters of the glacier:

$$\lambda=\frac{(1+\mu\nu)\beta\nu y_0-\alpha\beta}{3\alpha}. \\ c=\frac{(1+\mu\nu)\beta}{3\alpha} \quad (6)$$

For the studied Grosser Aletschgletscher: b_0 = 3900 m, μ = 10, ν = 0.1, α = 3 $m^{1/2}$, β = 0.007a^{-1}, L_0 = 22000 m [Oerlemans, 2008]. Respectively, c = 0.002 $a^{-1}m^{-1/2}$ and the characteristic time of the glacier length change, $\lambda^{-1} \approx$ 50 years.

Interannual values of E are not correlated [Dobrovolski, 1992]. Therefore, the autocorrelation function of the force $\eta(t)$ is approximated by equation (3) and we can assume that τ_η = 1 a. Because $\tau_\eta << \lambda^{-1}$, ΔE can be estimated as a δ-correlated random process (white noise). In this case, equation (5) can be interpreted as the Langevin equation (see above), describing a slow increment of the glacier length due to the summation of many "fast" changes of the altitude of the equilibrium line.

It is well known that the solution of equations (5 and 3) is a stochastic process, and its variance is given by:

$$\sigma^2_{\sqrt{\Delta L}}=\frac{\tau_r\sigma^2_\eta}{\lambda}\left(1-e^{-2\lambda t}\right), \quad (7)$$

if $t \ll \lambda - 1$, expression (7) is reduced to:

$$\sigma^2_{\sqrt{\Delta L}}=2\tau_r\sigma^2_\eta t. \quad (8)$$

The corresponding spectral curve displays a red-noise continuum. On the other hand, when $t \gg \lambda^{-1}$, the variance is defined as:

$$\sigma^2_{\sqrt{\Delta L},st}=\frac{\tau_r\sigma^2_\eta}{\lambda}, \quad (9)$$

where $\sigma^2_{\sqrt{\Delta L},st}$ is constant, characterizing the steady state conditions. The corresponding spectral curve displays a white-noise continuum.

Returning to the Grosser Aletschgletscher variability, let us calculate variance of its length. Application of equations (7) or (8) requires information about the initial undisturbed state of the glacier. However, assuming that the glacier has been always in disturbed state, we will apply expression (9) estimating the upper limit of variance. In fact, the answer to the question posed in the paper (about the role of stochastic forcing) depends on the variance calculated by expression (9): can the real behavior of the glacier be statistically described using this value of variance?

RESULTS

In order to use expression (9), the equilibrium-line altitude (E_0) has to be determined. It depends on climate condition of the region, but establishing this function is a complex problem [Six & Vincent, 2014].

We will estimate E_0 using different techniques. Firstly, we can do it by taking into account the averaged adiabatic lapse rate in the atmosphere (~ 0.0065 K/m) (see [Oerlemans, 2008]). It is clear that decrease of temperature along the glacier does not necessarily coincide with the lapse rate value; however, it cannot lead to a large error. The equilibrium-line altitude changes were estimated based on the range of interannual standard deviations of temperature (typically it equals ~2 °C for the extratropical belt). Considering the fact that in many regions (e.g. the Alps) the contribution of variation of precipitation to the change of glacier length is practically the same as the contribution of temperature variations, E_0 was estimated to be of the order of 600 m.

Several studies have shown that the snowline altitude at the end of the hydrological year is a good indicator of the equilibrium-line altitude. Therefore, to estimate E_0, we calculated the budget of snow along the profile of the glacier. We used the output data from the UBRIS (University of Bristol, Great Britain) integrations for simulation of the current climate (50 years) [PMIP2 Data Base]. In order to calculate the glaciological values, the model data (2.5 × 2.5°) have been downscaled to the Bernese Alps territory. It was performed using the method of detalization, utilizing gridded information about height, tilt, and orientation of the slopes, type of the surface, and closing the horizon [Kislov, Surkova, 2009]. The equilibrium-line altitude was determined as the height where the difference between the precipitation sum during the accumulation season and the layer of melted water during the ablation season was closest to zero (delineation of the ablation and accumulation periods were carried out based on time when the temperature reaches 0 °C [12]). The value of E_0 obtained as standard deviation of a set of values calculated for 50 years of a numerical experiment was 160 meters. Of course, the accuracy of this downscaling procedure cannot be high. Therefore, this result should be treated with caution.

Data of the UBRIS model, detailed for the Bernese Alps territory, were again used for calculating E_0, but utilizing another method, developed previously for determination of the snow boundary position on plains [Kislov, 1994]. This method is based on the dimensional and similarity theory; the input values are monthly mean temperature and water vapor pressure. The value of E_0 obtained as standard deviation of a set of values of boundary height calculated for 50 years of a numerical experiment was 300 meters.

Finally, the data of the UBRIS model, detailed for the Bernese Alps territory, were again used for calculating E_0, but utilizing another method, where the equilibrium-line altitude was calculated based on a regression equation where independent variables are precipitation sum during the cold season and the mean altitude of the isotherm 0 °C during the warm period. This equation was solved based on monitoring of 52 glaciers of the mid-latitude zone [Greene et al, 1999]. The value of E_0 was calculated at 700 meters.

Despite the fact that all the methods are not very reliable, the estimation of E_0 (was estimated to be of the order of a few hundred meters) is credible, because it was obtained using four very different independent approaches.

DISCUSSION

The above-mentioned values of E0 allow to determine the range of variance σ_η^2 = 0.1 – – 1.0ma–2. This also allows us to calculate (using expression (9)) $\sigma^2_{\sqrt{\Delta L},st}$ (it is, practically, the assessment of standard deviation: $\sigma_{\Delta L,st}$). It was equal to 5–50 m (depends on the σ_η^2 range).

Now let us consider the Grosser Aletschgletscher observation data [Fluctuations...]. Over the last 50 years, the glacier has been retreating (reduction of its length is about 1.5 km (Fig. 1)) exceeding the internal variability (this is typical of many glaciers [Reichert et al, 2002]). Apparently, the reason of such behavior is progressive warming observed in the Alps region during several decades (actually, since the middle of the XIX sentury, since the last phase of the Little Ice Age). The temporal behavior of the glacier has been gradually adapting to long-lasting dynamics of external forcing. Possible future glacier retreat is evaluated in [Jouvet, 2011].

Fig. 2 shows variation of glacier length and temporal behavior of the North Atlantic oscillations (NAO) index during the cold season [Climate Prediction Center]. NAO is an important indicator of cyclonic activity in Western Europe, which can be considered an indicator of interannual variability of meteorological regime. For the Alps region, high NAO indexes denote the situation when winter precipitation is below the normal value while the temperature anomaly is positive; therefore, in such a year, glaciers retreat [Beniston, 2006]. However, positive NAO indexes more often coincide with positive anomalies of glacier length (coefficient of correlation equal to 0.3) (Fig. 2), but the opposite situation is also common. Such asynchronism between forcing and response is an indirect consequence of the Brownian theory.

Let us compare the Grosser Aletschgletscher length deviations from the trend line and the values of standard deviation calculated by expression (9) (using $E_0 \approx$ 450 m – the arithmetical average of all mentioned estimates). It is clear that the deviations fall within the range ±2std (Fig. 2). The closeness of the calculated theoretical $\sigma_{\Delta L,st}$ to the range of empirical fluctuations could indicate the correctness of the Brownian approach.

Thus, based on the general laws on temporal dynamics relating to massive inertial objects, the observed interannual changes of the Grosser Aletschgletscher length could result from the accumulation of small heat/water flux anomalies. Under these conditions, the Grosser Aletschgletscher is represented by a system undergoing a random walk. Its

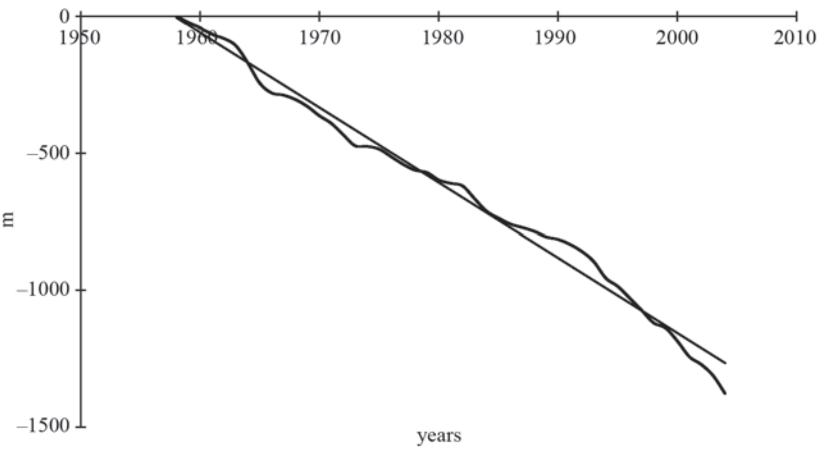

Fig. 1. Observation changes of the Grosser Aletschgletscher length (1958 to 2004, solid line) and a linear trend line (straight line)

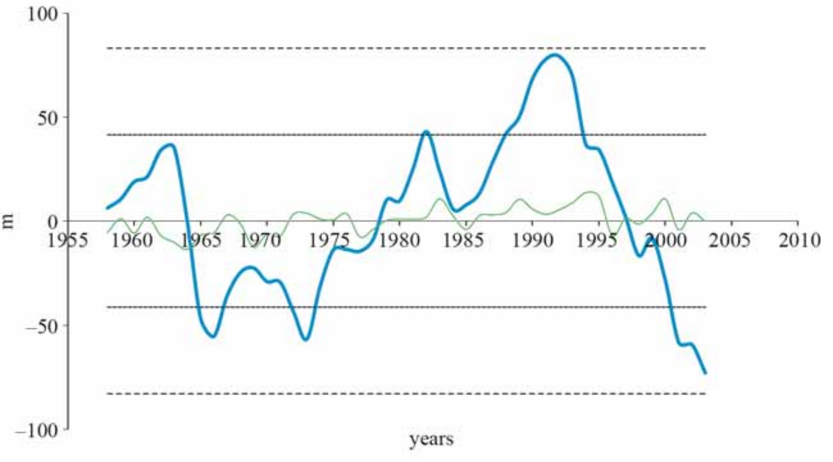

Fig. 2. The Grosser Aletschgletscher length deviations (blue line) from the trend line (*L-Ltrend*), theoretical assessment of the range of length variations (±1 std (small dots) and ±2 std (dashed lines)), and NAO index variations (green line) (values increased 10 times)

irregular interannual fluctuations do not practically correlate with external events.

CONCLUSION

The model was developed to investigate the response of glaciers to climatic forcing. This model is based on the so-called "minimal model" of the dynamics of mountain glaciers. We used as example the Grosser Aletschgletscher. The theoretical results have been interpreted from the stochastic viewpoint, despite the fact that the original model is deterministically based on the physical law of conservation of water mass.

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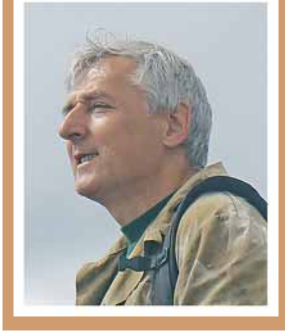

Alexander V. Kislov graduated from the Lomonosov Moscow State University in 1975 and received his Master's degree. Since January 1996, he has been Professor, Faculty of Geography, Lomonosov Moscow State University. The focus of his research is climate change. Main publications: Le changement climatique: Europe, Asie Septentrionale, Amerique du Nord.

Polina A. Morozova graduated from the Lomonosov Moscow State University in 2010, and received her Master's degree. Since January 2011, she has been Scientific Researcher at the Institute of Geography RAS. The focus of her area of interest is climate change. Main publications: Modeling of the Caspian Sea level changes during the last glacial maximum.