RESEARCH PAPER

OPTIMAL BANDWIDTH FOR GEOGRAPHICALLY WEIGHTED REGRESSION TO MODEL THE SPATIAL DEPENDENCY OF LAND PRICES IN MANADO, NORTH SULAWESI PROVINCE, INDONESIA

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ABSTRACT. Bandwidth plays a crucial role in the Geographically Weighted Regression modelas it affects the model's ability to describe spatial dependencies. If the bandwidth is too large, the model will be similar to a normal regression model. Conversely, if it is too small, the model will be too rough. Bandwidth can be selected in several ways, e.g. manually determined by experts or using Akaike Information Criteria, Cross-Validation, and Lagrange Multiplier methods. This study offers an alternative approach to choosing bandwidth based on the covariance function representing a linear combination between the Bessel and Gaussian-Type functions. We applied this function to analyze the land price in Manado with four infrastructure accessibility variables, such as accessibility to government offices, education facilities, shopping centers, and healthcare facilities. Therefore, the proposed method is different from the index methods (AIC and CV) which have been used by other researchers. The results showed that the non-parametric covariance function provides a smaller bandwidth than conventional methods, specifically Akaike Information Criteria and Cross-Validation. In addition, the value of R^2(adjusted) given by the covariance function is greater than the one given by the proportional method. This means that the optimal bandwidth obtained using the covariance function is more suitable to explain the land price in the city of Manado.

KEY WORDS: bandwidth, covariance function, land price, infrastructure

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INTRODUCTION

The modelling capabilities of Geographically Weighted Regression (GWR) are greatly influenced by the choice of bandwidth (geographical distance) and kernel function. The selected bandwidth limits the data (in this case, the location) to be within the bandwidth range. If the bandwidth is large, more data will be included in the analysis. The obtained GWR model will be smooth and closer to a simple regression model. Conversely, a small bandwidth will limit the data to a small sample, which in its turn will result in a roughmodel. Therefore, choosing the suitable bandwidth is an open problem which is very interesting to study. Bandwidth selection is usually done through several methods, e.g., manually by field experts or using Akaike Information Criteria (AIC), Cross-Validation (CV), and Lagrange Multiplier (LM) methods. The resulting bandwidth also varies, which makes determining bandwidth a challenging problem.

Because of the high use of GWR, several advanced studies have been carried out focusing onthe bandwidth selection, particularly using AICc and CV, which are very common data-driven methods that are sensitive to outliers. Leung et al. (2000) proposed to use the goodness of fit test and stated that bandwidth represents the degree of freedom of the GWR residue. Brunsdon et al. (2000) formed a bandwidth vector using the backfitting algorithm. Farber and Paez (2007) reduced bias by modifying the CV procedure. Meanwhile, Leong and Jue (2017) performed iteration procedures as well as solved numerical iteration problems. The use of correlograms to determine the geographical distance (bandwidth) was also implemented by some researchers, for example, spatial correlograms with index morans were used by Abebe et al. (2015) to measure wheat field landscapes in Ethiopia, or by Hetmanski et al. (2010) who measured the effects of pigeon habitat in 33 cities in the Pomeranian province, Poland. In addition, a spline correlogram was also used by Liu and Wemberly (2015) to measure the influence of climate and landscape on the spread of fires in the western United States from 1984 to 2010, and by Bjornstad et al. (1999) to measure the dynamics of tree mice and field mice in northern Japan during 1962-1992. However, correlograms are indexes that do not have the form of a covariance function as a condition of validity. Weku et al. (2019b) used a correlogram with a covariance function that appropriates the validity of definite positive to determine geographic distance.

In practice, bandwidth is a measure that determines the geographical distance between one location and other locations. For example, the fluctuation of land prices is influenced by the proximity of the studied location to urban centers that have a complete infrastructure. The closer a location is to the infrastructure center, the higher will be the land price, and, on the other hand, the land price will be lower when it is located far from the infrastructure. According to Manganelli et al. (2013),the land price can be significantly influenced by many factors. Apart from several general aspects (e.g. macroeconomic conditions, profit security, taxation), it is also affected by intrinsic and extrinsic factors. Extrinsic factors in urban areas are closely related to the spatial distribution of goods and needs. Because of that, the existence of infrastructure (access to public services and transportation, the existence of basic commercial services, etc.) and the environment (social context, noise level, clean water, etc.) in the spatial context become very important for marketing experts. As a result, extrinsic factors play a very large role in determining the geographical distribution of land prices. Land prices in Indonesia are not set by the government, the market price of land adheres to a closed market and is only known to the seller and the buyer. The determination of land prices carried out by the government depends on the Tax Object Sales Value (Nilai Jual Objek Pajak-NJOP) which is a reference for assessing land prices. However, this NJOP often does not reflect the actual land price.

The land price in Manado is considered very attractive because in recent years there has been an upward trend along with the development of the city's public service infrastructure. In particular, public facilities and infrastructure in the downtown Central Business District (CBD) include government offices, education and health services, several crowd centers, etc. The economic theory of classical urban land states that distance is one of the most important factors that determine the land price. Along with the increasing land prices inside the CBD where workers and business people gather, land prices around the CBD also increase as the distance to the destination decreases, resulting in higher accessibility and reduced travel costs. So, it is not surprising that the land price in the vicinity of Manado's CBD has increased due to the accessibility of public service infrastructure. To explore the idea of land prices, it is very important to understand their relationship with the accessibility of public service infrastructure.

Some studies on land prices and external factors have been done before, for example, Cellmer et al. (2014) used 5 variables (land use, ownership status, infrastructure, geometric configuration, and noise level), Du and Mulley (2012) used 3 variables (property data, transportation access-infrastructure, and socio-economic data), while Jiao and Liu (2012) used 3 variables (economic index, location, and infrastructure). In general, extrinsic variables that influence land prices were used in these studies, including the proximity of infrastructure locations. Manganelli et al. (2013) stated that extrinsic factors in the form of infrastructure, accessibility of specificpublic services, and availability of basic commercial services are very important in determining land prices. Grace and Saberi (2018) measured the accessibility of infrastructure for a certain location using spatial autocorrelation (Moran's I) followed by an Ordinary Least Square model. Chen (2018) studied land price differentiation and its factors in Guangdong, China using Exploratory Spatial Data Analysis (ESDA) and GWR methods. Apart from the mentioned factors, topography (e.g. valleys, slopes, earthquake traces, etc.) also has a significant influence on land prices (Kok 2011). However, it was not considered in this study.

Because determining bandwidth is very important to analyze the geographical distance correlation of land prices, bandwidth optimization became the main objective of the study. After that, the obtained bandwidth wasused to model the land price in Manado. This means that the GWR model is based on a correlogram matrix (covariance function) which is not the same as the matrix used so far.

MATERIALS AND METHODS

The use of accurate samples and data is very important for carrying out statistical analysis and interpreting the results. Inthis study, land prices in the city of Manado are expected to correspond to extrinsic factors, particularly to the accessibility of infrastructure from the observation sites. This land price model was developed using 150 sample locations.

RESEARCH AREA

This research was carried out in Manado city which is the capital of North Sulawesi province (Indonesia). Geographically, Manado is located on Manado Bay and surrounded by mountainous terrain. Based on BPS (Central Bureau of Statistics) data (Statistics of Manado Municipality, 2018), in 2017 the city had a population of430133 in habitants. The large population in Manado leads to a high population density. With an area of 157.26 km^2, the population density reaches 2736 people/km^2. The city of Manado is located at the edge of the northern peninsula of the island of Sulawesi, at 124°40' - 124°50' East and 1°30' - 1°40' North.

Fig. 1. Location of the study area in Manado City, North Sulawesi Province, Indonesia

ACCESSIBILITY

Location accessibility is a very important element of external factors that affect land prices. In a very general sense, the word accessibility refers to the ease of reaching potential destinations from certain locations using certain transportation systems. There are various approaches to measure accessibility depending on the objectives of the research. Continuous measurement (Hansen/Gravity Accessibility Measurement) is known as a robust approach to measure general accessibility for a particular service. For example,for education accessibility,the closest distance can be usedas a potential approach to measure accessibility because each student generally wants to live in areas that are near educational facilities.

Because of this, four accessibility variables that are expected to affect land prices in the city of Manado were used, i.e. the accessibility of government offices, educational accessibility, shopping center accessibility and hospital accessibility. The locations for the four variables are shown in Figure 2 (b). For the Government Office Accessibility, there were two locations of government offices used in this study, i.e. the Governor's office and the Mayor's office (red). For Education Accessibility, there are three major tertiary education locations in the city of Manado, including two public and one private institution (blue). For the Accessibility to Shopping Centers, there are 8 shopping center locations divided into modern shopping centers and traditional shopping centers (green). For Hospital Accessibility, there are 7 hospitals where 4 hospitals are owned by the government and 3 hospitals are owned by private parties (yellow). All 150 studied land locations are close to the highway, as well as the locations of infrastructure corresponding to the four variables that are considered to affect land prices.

GWR MODEL

GWR is a technique that brings the framework from a simple regression model to a weighted regression model (Fotheringham, et al. 2002). As a result, it produces a local linear regression model that generates a local parameter estimator model for each point or location where the data is collected. In a GWR model, the dependent variable is predicted by an independent variable whose respective regression parameter values depend on the location where the data is observed. AGWR model can be written as follows:

$$y_i = \beta_0(u_i, v_i) + \sum_{k=1}^{P} \beta_k(u_i, v_i) x_{ik} + \varepsilon_i \tag{1}$$

Where y is the estimated value of the dependent variable for observation i, x_{ik} is the value of the k^{th} variable for i, (u_i, v_i) is the coordinate location of i, β_0 is the intercept, $\beta_k(u_i, v_i)$ is the regression parameter of i, ε_i is the error term with iid assumption.

The purpose of GWR is to estimate these parameters for each independent variable X and for each geographical location i. The GWR estimation procedure is as follows: (i) drawing a circle of a given bandwidthh around one particular location i (center), (ii) calculating the weight for each neighboring observation according to its distance to the center, and (iii) estimating the model coefficient using the least-squares regression. As a result,the estimation of each row uses the equation:

$$\hat{\beta}(i) = (X^T W(i) X)^{-1} X^T W(i) Y \tag{2}$$

where X is the data matrix of the independent variable, Y is the vector of the dependent variable, $W(i)$ is the geographical weight matrix for the center $i = diag\ [w_{i1}, w_{i2}, \ldots, w_{in}]$.

W_i can be written as $W_i = f(d_{ij}, h)$, where $f(.)$ is a spatial kernel function, d_i is a vector of the distance between the center i and all neighbors, and h is the bandwidth or decay parameter. The fixed spatial kernel function that was used in this study to describe the geographical weight matrix W is the Gaussian descending distance kernel function:

$$[K(r)] = exp(-0.5(d_{ij}/r)^2) \tag{3}$$

The fixed kernel function assumes that the bandwidth in each centre i is constant in all observation areas. If locations i and j intersect, then $w_{ij}=1$, where w_{ij} is decreasing following the Gaussian curve along with the increasing d_{ij}. The weight is not zero for all data points, no matter how far all the points are from the center i (Fotheringham et al., 2002).

BANDWIDTH SELECTION: COVARIANCE FUNCTION

To estimate bandwidth, we propose using a covariance function. The used covariance function must be valid, meaning that it should have positive definite characteristics (Golinski, 2018). Weku et al.(2019a) showed that the exponential and Bessel functions are valid and positive definite. The Gaussian-type covariance function is given by:

$$C_{Gau-type}(h) = \exp\left(-\frac{h}{r}\right)^m \tag{4}$$

for m=1, the equation becomes an exponential function and can be rewritten as

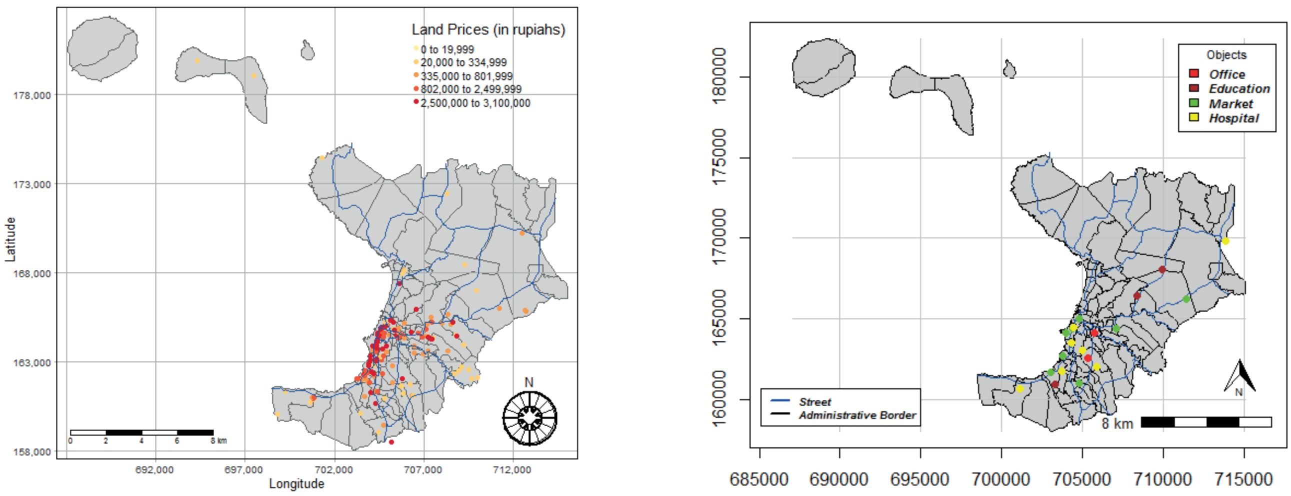


Fig. 2. (a) The distribution of land prices in the study area, (b) The locations of government offices, universities, shopping centers and hospitalsas independent variables

$$C_{\exp}(h) = \exp\left(-\frac{h}{r}\right)^{1} \quad (5)$$

The Bessel covariance function can be written as

$$C_{bessel}(h) = \sum_{k=0}^{\infty} J_0\left(\frac{kbh}{\omega}\right)^{1} \quad (6)$$

Based on the principle of linear combination, the Bessel-Gaussian type covariance function can be written as follows:

$$CF = \sum_{k=1}^{2} J_0\left(\frac{kbh}{\omega}\right) exp\left(-\left(\frac{h}{\omega}\right)^{1}\right) \quad (7)$$

where J_0 is a Bessel function, k is a base, scalar b represents nodes of the wave, h is the longest distance of locations, while ω is a parameter related to the distance (i.e. 1/3.h).

The following steps will show that both functions are valid. We will show that the exponential function is definite positive(valid) as shown by Schoenberg (only the first proof).

Proof. Taking corollary in Schoenberg (1937), it is stated that if $\Phi(x)$ is homogeneous and such that $e^{-\Phi}$ is positive definite then $e^{-\Phi\gamma}$ is also positive definite for $0<\gamma<1$.

Furthermore, we know that e^{-x^2} is a positive definite by virtue of its formula and properties. Consequently $e^{-|x|^{2\gamma}}$ is positive definite for $0<\gamma<1$.

As we know, valid positive definite functions are finite. Weku (2019b) makes it clearer by showing that the Bessel function is positive definite (valid) because it has an exponential upper bound. Based on the Weierstrass principle, because the exponential function is positive definite, the Bessel function is also positive definite.The relationship between the Bessel functions bounded by the exponential function can be written as follows:

$$|J_v(x)| \le \frac{1}{v!}\left(\frac{|x|^v}{2}\right) exp\left(\frac{|x|}{2}\right)^2$$

It is known that the exponential and Bessel covariance functions are valid and meet definite positive conditions. This means that the covariance function can be used as a correlogram to determine spatial geographic distances of land prices in Manado.

RESULTS AND DISCUSSION

The results and discussion section can be divided into two parts, the first part focuses on selecting bandwidth from several existing methods and in the second part, the optimal bandwidth is used in GWR modeling. Three methods for determining bandwidth were considered in this study, i.e. AICc, CV and Covariance Function (proposed method). The equations for calculating bandwidth with the AICc and CV methods can be written as follows (Mennis, 2006):

$$AIC_c = 2n\log_e(\hat{\sigma}) + n\log_e(2\varphi) + n\left\{\frac{n+tr(S)}{n-2-tr(S)}\right\}$$

$$CV = \sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2$$

The results of calculations using the Covariance Function are displayed in Figure 3, where the value of covariance is plotted against distance. The intersection between the covariance curve and the distance axis represents geographical distance.The first intersection with the distance axis is the optimal bandwidth because it illustrates the strength of the correlation around the origin. After that, the correlation weakens which does not reflect the strength of the correlation.

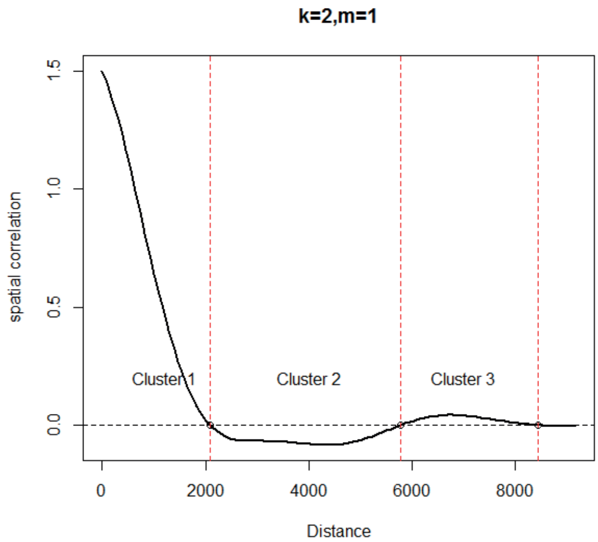


Fig. 3. The Optimal Geographical Distance of land prices in Manado

The three methods for calculating bandwidth were implemented using R software. The results of the bandwidth calculation are presented in Table 1. The largest bandwidth was found by AICc while the smallest one was found by the Covariance Function. Three parameters were calculated to determine the appropriate bandwidth for spatial dependence of land prices, namely R2 (adjusted), R2 and AIC. The appropriate model was then selected based on the largest R2 (adjusted) and R2 values as well as the smallest AIC value.

Table 1 presents three different bandwidth estimates, where the bandwidth of the covariance function is the smallest while the bandwidth of AICc is the largest. The bandwidth of 2091.351 meters obtained from the Covariance Function calculation is the optimal bandwidth because it has the largest coefficient of determination R2 (adjusted), which is equal to 0.5022. Conversely, the bandwidth values provided by AICc and CV with low coefficients of determination (0.2571 and 0.2614, respectively) have a low ability to explain the land prices. Even though the AIC values are almost the same for the

Table 1. The assessment of various bandwidth selection methods

Assessment	Bandwidth Selection Methods		
	Covariance Function	AICc	CV
	2091.351 meters	5452.273 meters	4736.107 meters
R2(adjusted)	0.5022	0.2571	0.2614
R2	0.3786	0.3033	0.3130
AIC	4579.152	4554.265	4554.101

three bandwidths, we can state that the model that uses the bandwidth of the covariance function reproducesthe actual land price dataand out performs AICc and CV methods. As a result, we chose a bandwidth of 2091.351 meters to model land prices using GWR.

ESTIMATED COEFFICIENT OF THE GWR MODEL

GWR software provides results in the form of information on spatial variation. Table 2 illustrates descriptive statistics generated by GWR with a bandwidth of 2091.351 meters. The values calculated for quartiles represent the variability of land prices witheach variable.

SPATIAL CHARACTERISTICS OF THEESTIMATED COEFFICIENT

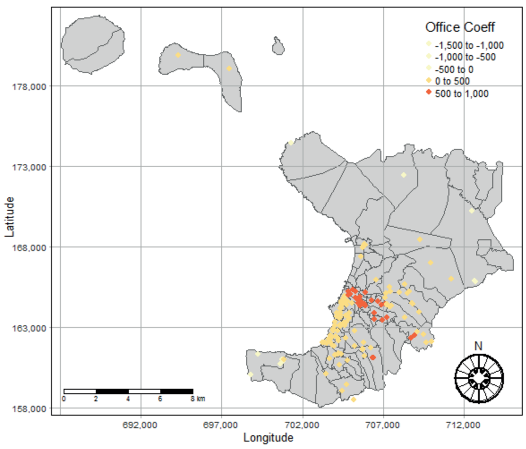
The results of the GWR model forthe four variables (accessibility to government offices, universities, market centers, and hospitals) were mapped as shown in Figure 4 to see spatial variation and analyse the effects of infrastructure accessibility factors on land prices. The coefficients of the GWR model estimated using the fixed kernel function are concentrated in the downtown area.

Figure 4(a-d) shows the results of parameter estimation using different colors and gaps between variables. Values of local parameters are colored in such a way that positive values are represented in red, negative values are represented in white and yellow corresponds to values around zero. It is very clear from the map that all parameters indicate spatial variation, especially the government variable.

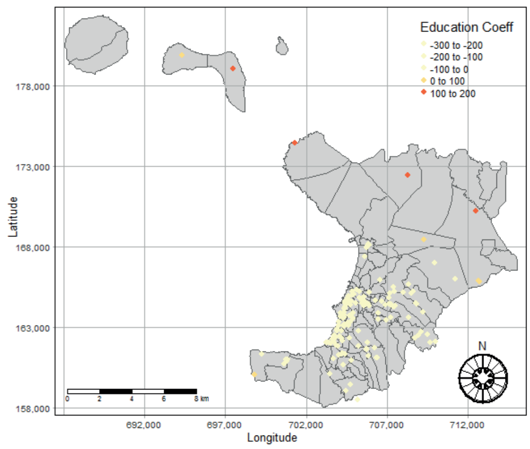
In Figure 4a, which shows local parameter estimates for the Government Office variable, it is very clear that there is spatial variation in land prices throughout the area. The coefficient will be positive in most areas that are centralized and negative in some of the outermost observation areas. This means that land prices are strongly influenced by the accessibility of the governor's and mayor's government offices. The geographical center shows that local parameters

Table 2. Results ofGWR coefficient estimation with a bandwidth of 2091.351 meters

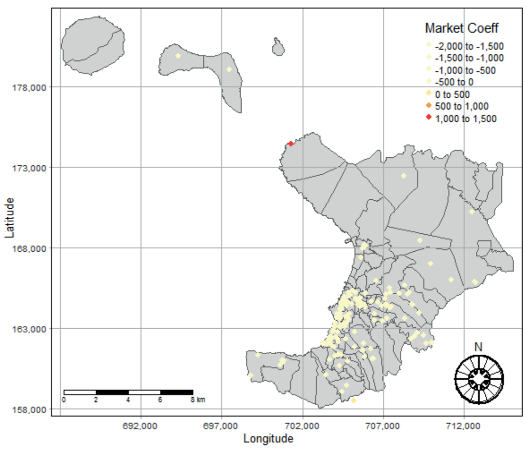
	Min.	1st Qu.	Median	3rd Qu.	Max.
Intercept	-178403	1942007	2032163	2300653	2585733
Gov	-1024.790	229.355	428.058	491.648	516.030
Univ	-270.658	-140.124	-86.586	-43.555	164.040
Market	-618.881	-531.782	-464.042	-424.350	1413.650
Hospital	-558.326	-443.590	-409.258	-361.990	772.020



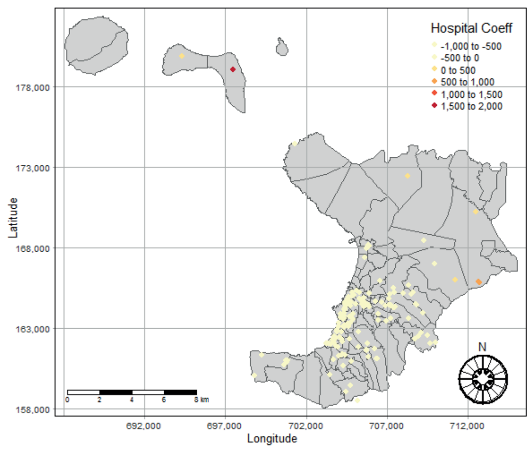
a. map of parameter estimates with variable Office



b. map of parameter estimates with variable Education



c. map of parameter estimates with variable Market



d. map of parameter estimates with variable Hospital

Fig. 4. Map of parameter estimates for each variable (Government Office, Universities, Market Centers and Hospitals)

are significant positive indicating that proximity to the Government Office will increase the price of land in urban centers. In the outermost region, parameters are significant negative even in small amounts. The negative parameter value meansthat the low accessibility of the Government Office leads to a negative increase in land prices in these locations.

This means that land prices in each location are strongly influenced by the presence of the governor's and mayor's government offices. The land that is closer to the office is more expensive, while the price of the land located far from the office center is lower due to the lack of government facilities.

Meanwhile, Figures 4b, 4c, and 4d show that there are clear spatial variations in local parameter estimates for the Education, Shopping, and Hospitals variables through out the observation area. The general pattern is the same, higher access to these facilities (resulting in decreasing travel time due to shorter distances) will result in increased land prices. However, there is a large area that is shown in Figures 4b, 4c, and 4d where local parameters are significant negative. This area covers a large part of Manado city and approaches the city center. All regions with a negative relationship between the three distance variables andland prices are inverse to the regions that have access to Government Office facilities shownin Figure 4a.

In this study, it was shown that bandwidth estimated using the covariance function gives very good results and outperforms the traditional methods (AICc and CV). It was found that the location of government offices, in this case, the governor's office and the mayor's office, has a significant influence on the increase in land prices. This suggests that residents of Manado city are still dependent on government services. On the other hand, land prices are not too dependent on education, health, and market service facilities because there is a variety of options that are spread evenly through out the city.

As explained earlier, the government does not perform open regulation of land prices, resulting in the sale and purchase of land carried out in a closed manner between the seller and the buyer. Research on land price prediction using the GWR model has provided a new approach that can assist the government in determining land prices based on the proximity of the observed land locations to various facilities. In this approach, land prices can be adjusted at any time depending on the variables used in the model. As a result, the government and community will have the same understanding in determining the land price so that people willno longer need to make closed sales. The results of predictive mapping using contour maps present land prices that can be used as an open reference for the community in addition to the NJOP provided by the government. Currently, the land price in a location varies according to the assessment of each person as everyone can sell their land based on their assumptions. As a result, land prices become uncertain because they are not set openly by the government, which can lead to the practice of selling land in discriminately by speculators.

CONCLUSIONS

Determination of optimal bandwidth is fundamental and crucial formodeling the spatial dependence of land prices using GWR. This study has shown that using the Covariance Function to determine bandwidth provides a large coefficient of determination R2 (adjusted) when compared to the bandwidth calculated using the AICc and CV methods.This means that the bandwidth produced by the Covariance Function is suitable for modeling land prices in Manado.

Determining land prices without taking into account their spatial dependence can cause errors. The development of spatial statistical techniques based on Geographically Weighted Regression can help to overcome these errors and provide a better analysis of land prices. In this study, the GWR technique was applied to analyze land prices in the city of Manado and determine the homogeneous locations.The results provided by the local GWR model revealed the relationship between thespatial variation ofland prices and the variables usedin the model. Mapping of the results showed that accessibility of Universities, Market Centers and Hospitals affect land prices with negative parameter estimates, while Government Office accessibility has positive parameter estimates. Determination of the appropriate model to describe land prices can also be done using the AICc assessment, which shows that the Hospital variable has the greatest influence on the price of land in the city of Manado. This selection is based on the smallest AICc value, which is followed by the Government Offices, Universities and Market Centers variables.

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