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RISK ASSESSMENT BASED ON MATHEMATICAL MORPHOLOGY OF LANDSCAPE MODELS (CASE STUDY OF THERMOKARST PLAINS)

ABSTRACT. The paper presents a new approach for risk assessment of impact of thermokarst processes on engineering structures; it is based on the methods of mathematical morphology of landscape. The paper presents the results of the investigation of irregular (non-circular shape) thermokarst lakes. Remote sensing images of reference plots with thermokarst lakes were digitized; then, theoretical assumptions were applied and the simulation results were compared with empirical data to prove convergence. The analysis of the obtained results showed general agreement of empirical and theoretical data

KEY WORDS: risk assessment, remote sensing, landscape pattern analysis, mathematical morphology of landscape

INTRODUCTION

One of the urgent modern tasks in assessment of the risk associated with development of exogenous geological processes. The most common risk assessment parameters are:

- The probability of some degree of damage to engineering structures from exogenous process.
- The average risk – the mathematical expectation of losses (e.g., area or length) of an engineering structure by a hazardous exogenous process.

- The probability distribution of damage of an engineering structure by a hazardous exogenous process.

Risk assessment is the subject of many studies [e.g., Assessment and Natural Risk Management, 2003; Yelkin, 2004; Ivanov, Dulov, Kuznetsov, et. al., 2012], however, the task is still urgent. The difficulty is associated with the fact that processes, which allow assessing the probability and the size of damage are poorly studied. In the statistical approach, it is difficult to obtain a large volume of statistical data on damage to structures, especially considering every type of physical and geographical conditions that an engineering structure encounters. The time required to obtain such data is comparable with the time of operation of an engineering structure, however risk assessment is needed at the design stage of construction. Therefore, this approach is not very promising.

Previously, the possibility and the ways of use of morphological structure models in engineering structures risk assessment has been demonstrated for a number of genetic types of areas [Victor, 2006; Victorov, 2007, Victor, Kapralova, 2011]. However, the problem was solved for the simplest case, when the foci of the processes have a circular shape, and, thus, the solution was applicable for a limited number of situations.

OBJECT AND METHODS OF THE STUDY

The goal of the study presented herein is to demonstrate the solution to a problem of risk assessment using morphological structure models for a number of genetic types of the territories with the irregular shape foci of hazardous processes. The research was conducted using thermokarst lake plains as an example (Fig. 1).

The investigated type of terrain represents a sub-horizontal undulating surface with predominance of tundra vegetation, and with interspersed thermokarst lakes randomly scattered across the plain. Initially, emerging foci of thermokarst processes (lakes) are characterized by an almost round shape, but in the process of development, they tend to merge and, thus, the shape of the foci may differ significantly from a circular (Fig. 2). It is specifically this last factor that creates significant challenges in risk assessment.

In the process of development, the foci of thermokarst processes may undergo the following stages:

1. In the emergence of the primary focus, the major factor seems to be the accumulation of a relatively large water layer in depressions [Perlstein, Levashov, Sergeev, 2005];

2. The expansion of the focus (thermokarst lake) due to the thermoabrasive impact; its speed depends on many random factors (average air temperature, permafrost ice content, soil composition in the vicinity of the lake, etc.);

3. Possible merging with other adjacent lakes.

The solution to the task of risk assessment may be based on the morphological structure model for a thermokarst lake plain [Viktorov, 1995; Viktorov, 2005, Victor, 2006]. Let us consider a thermokarst lake site, homogeneous in respect to its physical-geographical and, first of all, geomorphological properties. The model was based on the following assumptions:

1. The process of emergence of the primary depressions is probabilistic and occurs in non-overlapping areas (Δs) and



Fig. 1. A typical landscape of a thermokarst lake plain

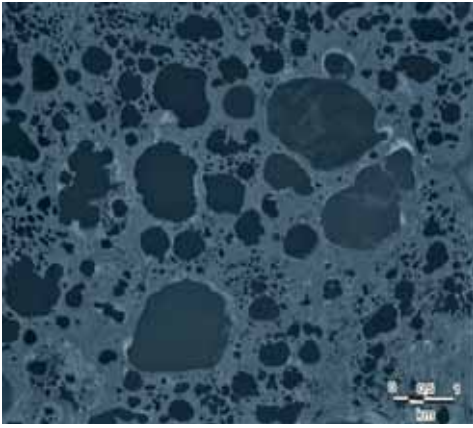


Fig. 2. A typical landscape of a thermokarst lake plain in remote sensing

at non-overlapping time periods (Δt); it is independent; the probability of emergence of a single depression is significantly greater than the probability of emergence of several depressions, that is,

$$p_1(t) = \lambda(t)\Delta s\Delta t + o(\Delta s\Delta t) \quad (1)$$

$$p_k(t) = o(\Delta s\Delta t) \quad k = 2, 3, \dots, \quad (2)$$

where $\lambda(t)$ means the density of newly appearing depressions per unit area at time t .

2. The growth in the size of the lakes due to thermoabrasion is a random process and is independent of the other lakes; in the course of development, the lakes can merge.

The assumptions seem to be reasonable since they are derived assuming the homogeneity of the study area and using existing ideas about the mechanism of the process.

This foundation provides the basis for a rigorous mathematical analysis of the assumptions in order to obtain consistent patterns of the structural features of a thermokarst plain [Viktorov, 1995; Victorov, 2005, Victor, 2006]. Thus, the distribution of thermokarst depressions (foci) at a randomly selected site meets the Poisson distribution, that is,

$$P(k, t) = \frac{[\mu(t)s]^k}{k!} e^{-\mu(t)s}, \quad (3)$$

where s is the area of the test site, $\mu(t)$ is the average number of depressions per unit area per unit time t . The density of depressions in general depends on time since the emergence of new thermokarst lakes is possible and, even in the absence of new lakes, existing lakes can merge.

The Poisson distribution of lakes is confirmed by our experimental data (Fig. 3) and other publications [Victorov 1995, 2006; Polishchuk, 2012; and others].

SIMULATION RESULTS

Let us compute the probability of damage of a linear structure of a given length (L).

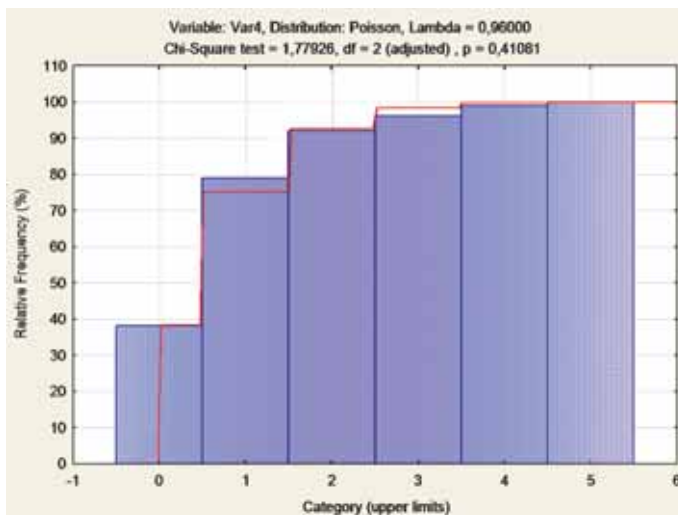


Fig. 3. Comparison of the theoretical and empirical distribution of the number of thermokarst lakes at a random site

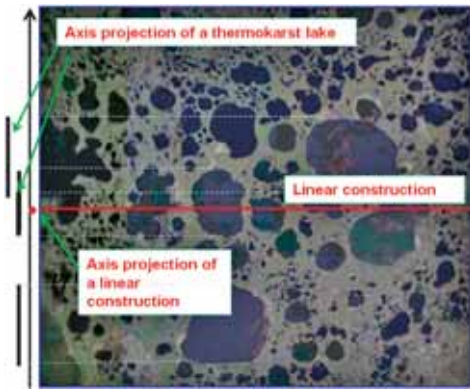


Fig. 4. Schematic representation of the procedure for probability assessment of damage to a linear structure (explanation in text)

First, we will consider the band of a finite width R (Fig. 4), at whose axis the linear structure is located. Consider a coordinate axis x perpendicular to the linear structure.

In the projection, this axis is represented by a point, the focus of thermokarst – by a segment, whose length corresponds to its projection on the axis. The damage of the linear structure is expressed as the intersection of the segment and the point. It may be easily demonstrated that the Poisson distribution of the foci at a site determines the Poisson distribution of their projections on the axis and, therefore, the equal probability of locations of the foci projections at given segments of the axis and their independence from each other. The probability (α) of the event when one, out of a given number, focus touches the linear structure (i.e., that the point corresponding to the projection of the linear structure will be inside the projection of the focus) depends on the above mentioned relations between the length of the focus projection and the width of a given band. This, considering the probability of different projection sizes, gives the following expression:

$$\alpha = \int_0^{2R} \frac{x}{2R} f_p(x, t) dx \quad (4)$$

where $f_p(x, t)$ means the distribution of the sizes of the foci projections at time t . The probability of the event when none of the foci touches the linear structure and

assuming that their number equals k and the centres are independent, is:

$$P^0(k, R, t) = (1 - \alpha)^k \frac{[2\mu(t)RL]^k}{k!} e^{-2\mu(t)RL}. \quad (5)$$

The level of safety (probability of damage) of a linear structure at a random number of the foci in the band may be obtained by summation over k and transition to the limit at unlimited extension of the given band ($R \rightarrow +\infty$)

$$P_{nl}^0(R, t) = \sum_{k=0}^{+\infty} (1 - \alpha)^k \frac{[2\mu(t)RL]^k}{k!} e^{-2\mu(t)RL} = e^{-2\mu(t)\alpha RL} \quad (6)$$

since

$$\lim_{R \rightarrow +\infty} 2\alpha R = \int_0^{+\infty} x f_p(x, t) dx = \overline{pr}(t), \quad (7)$$

where $\overline{pr}(t)$ means the mathematical expectation of the value of the foci projections at time t , therefore, after reduction the safety is

$$P_{nl}(L, t) = e^{-\mu(t)\overline{pr}(t)L} \quad (8)$$

Hence it is clear that the probability of damage of a linear object with the length L by at least one focus is

$$P_{dl}(L, t) = 1 - e^{-\mu(t)\overline{pr}(t)L}. \quad (9)$$

If a linear object has a shape of a curved line, this result may be considered a baseline. To assess the risk in this case, the curve should be approximated by a kinked line, and the probability of damage is computed for its segments according to (9).

Let us compute the probability of damage of an areal structure of a spherical shape with a given radius (l). The damage of the structure by the focus of the process at time t occurs in one of the two events:

- The center of the structure is within the focus contour,
- The center of the structure is outside the contour, but at a distance from the focus less than l .

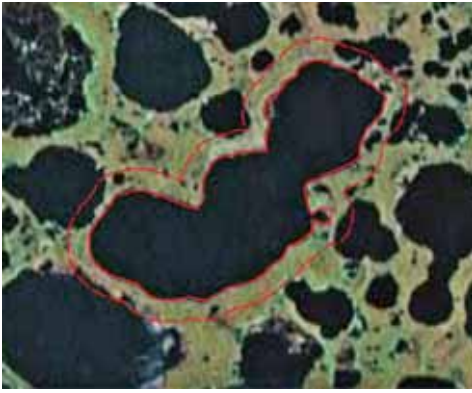


Fig. 5. Example of the l -buffer of a thermokarst focal point

The latter means that the center of the structure is in the l -buffer of the focus of the process. Let's denote the l -buffer of a shape by a set of points of the area outside its focus, but at a distance less than l (Fig. 5).

Thus, the probability of damage of the areal structure is equal to the product of the probabilities of the two above-mentioned events.

It is natural to conclude that the probability of the center of the structure to be within the contours of any focus considering the assumption for the focus area of its independence on the location and mutual non-intersection of the foci at a given moment, is

$$P_1(t) = \mu(t)s(t), \quad (10)$$

where $\mu(t)$ means the average number of the foci per unit area at time t ; $s(t)$ means the average area of one focus at time t .

To determine the probability of the second event, let us first review the circular area, where the structure is located, bounded by radius R . The probability of the event when the center of the structure will be within the l -buffer of the focus (i.e., the focus will "touch" the structure) is defined by the ratio of the area of the l -buffer of the focus and the entire area of the circular shape and considering the probability of formation of the buffers of the foci with different area is

$$\alpha(l, t) = \int_0^{\pi R^2} \frac{x}{\pi R^2} f_b(x, l, t) dx, \quad (11)$$

where $f_b(x, l, t)$ means the density of the distribution of l -buffer area of the focus at time t . Then, applying the algorithm for a linear structure presented above, evaluating the probability of the event that none of the foci touches the areal structure, assuming their number within the band is k , and eventually transitioning to a random number of the foci and extending the circular shape fairly, we get

$$P_2(l, t) = e^{-\mu(t)s_b(l, t)}, \quad (12)$$

where $s_b(l, t)$ means the average size of the l -buffer at time t given by the expression

$$s_b(l, t) = \int_0^{+\infty} x f_b(x, l, t) dx. \quad (13)$$

Thus, the overall probability of the circular structure with radius l in this model is

$$P(l, t) = 1 - [1 - \mu(t)s(t)]e^{-\mu(t)s_b(l, t)}. \quad (14)$$

The formula can be greatly simplified if the foci have a convex or a convex-concave

shape, but with curvature of at least $-\frac{1}{l}$, in

other words, if a concave site has smaller curvature than the boundary of the structure. In this case, we can analytically derive the following expression

$$s_b(l, t) = l\bar{p} + (t)\pi l^2, \quad (15)$$

where $\bar{p}(t)$ means the average perimeter of the focus at time t .

Accordingly, the probability of damage of the circular areal structure with radius l in this case is

$$P(l, t) = 1 - [1 - \mu(t)s(t)]e^{-\mu(t)[l\bar{p}(t) + \pi l^2]}. \quad (16)$$

The buffer-concept generalization allows us to find the expression for the areal-structure of not a circular, but of a complex form. In this

case, the buffer means the area surrounding the focus boundary, so that when it contains the center of the structure of given shape (C) and bearings, the focus touches the center of the structure. In these conditions, the buffer is a function of four factors: (1) the contour of the focus, (2) the contour of the structure, (3) the angle between the line connecting the focus and the center of the structure, and the diameter of the focus, and (4) the angle between the line connecting the focus and the center of the structure, and the diameter of the structure.

Overall, the analysis shows that the expression for the probability of damage of the areal structure is

$$P(l, t) = 1 - [1 - \mu(t)s(t)]e^{-\mu(t)s_b(C, t)}, \quad (17)$$

where $s_b(C, t)$ means the average size of the focus buffer at time t in relation to the engineering structure of a given shape (C).

Thus, the probability of damage of an areal structure at a given time depends on the average density of the foci, average area of a focus, and average area of the buffer of a structure of a given-shape. This set of parameters differs from the set of parameters in the computation of the probability of damage for a linear structure.

Solving the latter problem, we have actually solved the problem of the probability of merging of thermokarst lakes over time-period $[0, t)$. For this, it is sufficient to substitute the second focus for the engineering structure and to assume that its contours have some probabilistic distribution similar to the first focus. It is clear that the angle of the relative orientation of the foci (the angle between the diameters), due to the homogeneity of the conditions of the site, will have a uniform distribution on the segment $[0, 2\pi]$. As a result, the probability of the foci merging in the interval $[0, t)$ is

$$P_f(t) = 1 - [1 - \mu(t)s(t)]e^{-\mu(t)s_b(l, t)}, \quad (18)$$

where $f_{bi}(x, \alpha, t)$ means the density distribution of the area of a thermokarst

focus in relation to another focus, at a angle of the relative orientation at time t ; $s_{bi}(t)$; means the average area of the thermokarst focus buffer in relation to another focus at time t , given by the expression

$$s_{bi}(t) = \int_0^{2\pi} \int_0^{+\infty} \frac{1}{2\pi} x f_{bi}(x, \alpha, t) dx d\alpha. \quad (19)$$

From this formula, it is easy to obtain by differentiating the probability of the lake merging with some other lake at a given time-period $[t, t + \Delta t)$.

$$P_c(t, t + \Delta t) = \left[\frac{1}{1 - \mu(t)s(t)} \frac{d}{dt} \mu(t)s(t) + \frac{d}{dt} \mu(t)s_{bi}(t) \right] \times \Delta t + o(\Delta t). \quad (20)$$

DISCUSSION

The obtained expressions for linear structures were initially field-tested at standard plots (West Siberia). The testing of the expression for the assessment of the probability of damage for a linear structure was based on the following reasoning. Let us assume that we are at a given site before the start of the thermokarst process. Because the site is homogeneous, we do not have any reasons to prefer some specific location for a linear structure over another and the structure can be located, with equal probability, at any point at the site. The foci emerging in the future, visible now on the image, could have damaged or not the linear structure.

Considering this reasoning, the development of events in real conditions was simulated in the following way: using programming tools and the random number generator, we randomly placed a linear structure (segment) within a selected site (with already existing process foci) and,

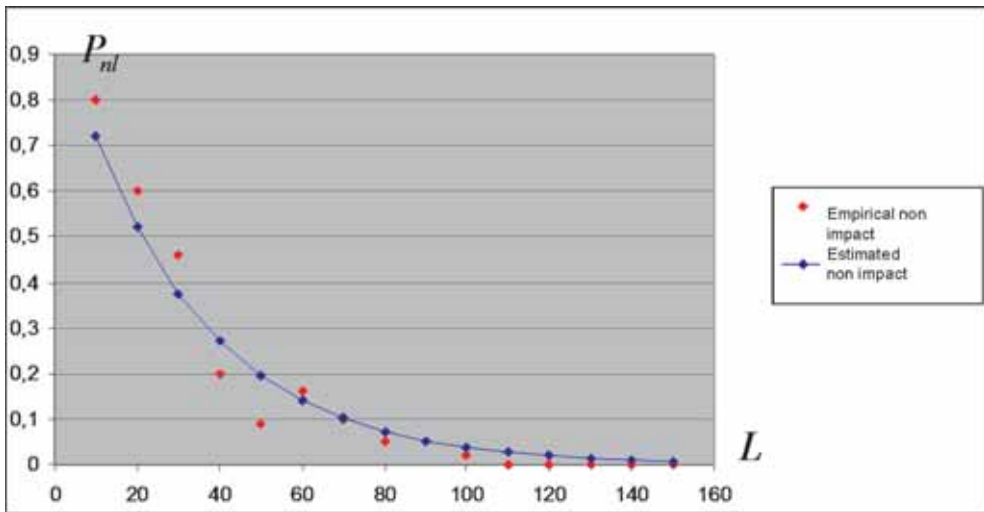


Fig. 6. Comparison of the theoretical and empirical probability of damage for linear structures

then, estimated the number of lines, not intersecting the foci. The obtained number of the linear objects not damaged by a focus (in fractions of the total number of cases) was compared with the computed numbers from expression (8).

The procedure was repeated for linear structures (segments) of different length. The test produced positive results (Fig. 6).

The model presented herein is the basis for the use of remote sensing in risk assessment, for example, in selecting a shape of alignment of a linear facility. Considering the information presented above, the procedure for a linear structure (for example) should contain the following basic elements:

- Forecast based on repeated computations of mathematical expectation of projections that appear over the duration of the structure functioning, on the axis perpendicular to the bearing of the linear structure (linear interpolation or linear interpolation of the mathematical expectation of the projection logarithm);
- Computation of the probability of damage using the obtained expressions (9).

In addition, it is necessary to account for already existing foci using the lognormal distribution [Victorov, 2007]; the parameters are defined from repeated surveys.

At the present time, design of a linear structure, for example, practically does not provide for quantitative assessment of the probability of damage of the structure over its lifetime by existing or appearing foci of exogenous processes. Karst risk represents an exception [Recommendations for Engineering Surveys..., 2012], however, strictly speaking, the recommendations refer to the risk of damage for 1 ha of area, and not to a linear structure; moreover, the recommendations do not account well for the size of the karst foci.

In our analysis, we have not used significantly the mechanisms of the thermokarst process, but only the fact that the distribution of the foci has the Poisson character and they change independently of each from one another. Therefore, the results obtained can be extended to other processes, where foci emerge and develop independently (for example, swamping, subsidence, etc.).

CONCLUSIONS

1. The expressions for risk assessment of linear and areal structures damage by processes with the Poisson distribution and with free-shape foci (thermokarst, collapsibility, karst, flooding, etc.) have been obtained.
2. The model of the morphological structure of thermokarst lake plains development

has been obtained; the model considers thermokarst lakes merging.

3. It has been demonstrated that zoning for risk associated with damage of linear and areal structures does not coincide in general case because the probabilities of damage risk depend on different parameters for foci of different hazardous processes. ■

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Alexey S. Victorov graduated from the Lomonosov Moscow State University; he obtained his PhD Degree in 1976 and his DSc Degree in 1988. He worked for VSEGINGEO and for the Scientific Geoinformation Center of RAS. In 1996, he, together with his colleagues, was awarded the State Prize in Science and Technology for his work on intergraded remote sensing used in design and operation of geotechnical systems. Since 2006, he has been Deputy Director for Research of the Institute of Geoecology of RAS. During his work at the Institute, he has been awarded the Prize of the Russian Federation Government (2003), the National Ecological Prize (2004), and the RAS Grigoriev's Prize (2006). His research is focused on creation of mathematical models of morphological structures formed by exogenous geological processes of different genetic types. This activity has led to development of a new trend in landscape science – mathematical landscape morphology. Significant results were obtained in the course of development of theory and methods of identification and interpretation of aerial and space imagery data to address tasks of engineering geology, hydrogeology, geoecology, and regional research on arid territories (Usturt, Tugay depression, Kyzyl-Kum, etc.). The results were summarized in numerous publications, including monographs “Mathematical Landscape Morphology” (1998), “Fundamental Issues of Mathematical Landscape Morphology” (2006), and “Natural Hazards of Russia” (2002, with co-authors).



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